Comparison of some methods for transportation problems

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Abstract

The transportation model is basically a Linear Programming Problem (LPP) that is able to be solved by regular simplex method. And then, its special structure allows the development which is called transportation technique that is more efficient computationally. We study and compare more efficient methods that have been developed to solve transportation problems.

1.Introduction

The transportation problem deals with a special class of linear programming problems in that the objective is to transport a homogeneous product manufactured at several plants (origins) to a number of different retails (destinations) at a minimum total cost. The total supply available at the origin and the total quantity demanded by the destinations are given in the statement of the problem. The cost of shipping a unit of goods from a known origin to a known destination is also given. Our objective is to determine the optimal allocation that results in minimum total shipping cost.

The transportation (or distribution) problem is significant for most commercial organizations that operate several plants and hold inventory in regional retails.

Let we consider the following transportation problem.

A firm has 3 factories *X*, *Y*, and *Z*. There are four major retails situated at *A*, *B*, *C*, and *D*. Average daily product at *X*, *Y*, *Z* is 120, 70, and 50 units respectively. The average daily requirement of this product at *A*, *B*, *C*, and *D* is 60, 40, 30, 110 units respectively. The transportation cost (in *Ks.*) per unit of product from each plant to each retail is given below:

Table (1.1)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20	22	17	4	120
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	60	40	30	110	240

The problem is to determine a routing plan that minimizes total transportation costs.

2. Mathematical Formulation

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In the standard interpretation of the model, there are *m*–supply points with the items available to be shipped to *n*–demand points. Specifically, plant *i* can ship at most S_i items, and demand point *j* requires at least D_j items. The S_i and D_j are fixed in reference to a stated time interval or planning horizon. The cost of shipping each unit from plant *i* to demand points *j* is c_{ij}. The objective is to select for, the duration of the horizon, a routing plan that minimizes total transportation cost.

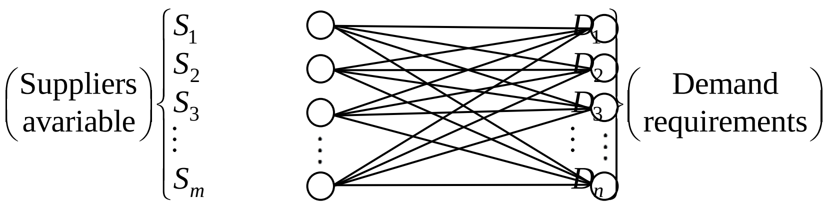

Figure (2.1) Network for Transport Problem

The mathematical description of the classical transportation problem is

Minimize: $$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}.$$

Subject to: $$\sum_{j=1}^{n} x_{ij} \le S_i \quad \text{for } i = 1,2,3,\ldots,m \text{ (Supply)}.$$

$$\sum_{i=1}^{m} x_{ij} \le D_j \quad \text{for } j = 1,2,3,\ldots,n \text{ (Demand)}.$$

$$x_{ij} \ge 0 \text{ for all } i \text{ and } j.$$

The network and technology table for this model is shown in Table (2.1).

In the mathematical description given in the above equations, the nonnegative quantity x_{ij} represents the amount of goods transported from plant *i* to all the demand points *j*. Observe that the sum of the shipments from plant *i* to all the demand points cannot exceed the available supply S_i. Similarly, the sum of the shipments to demand point *j* from all plants must be at least the demand requirement D_j. If the unit cost of producing an item defers from plant to plant then the cost is included in the determination of c_{ij}. If for physical or economic reasons a certain plant is inaccessible to a particular demand point, then the associated x_{ij} is eliminated or, if more convenient, the corresponding c_{ij} is defined to be arbitrarily large. To simplify the discussion, assume $c_{ij} \ge 0$.

In the sections that follow, we will concentrate on algorithms for finding solutions to transportation problems. Methods for finding an initial basic feasible solution are

(i) North–West Corner Rule,

(ii) Matrix Minimum Method.

3. North–West Corner Rule

The North–West Corner rule is a method to compute a basic feasible solution of a transportation problem, in which the basic variables are selected from the North–West Corner.

3.1 Example

Consider transformation Table (1.1).

Plant	Retail Shop				Supply
	A	B	C	D	
X	20	22	17	4	120
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	60	40	30	110	240

The economic problem is to distribute the available product to different retail shops in such a way so that the total transportation cost is minimum.

Starting from the North–West Corner, we allocate min (120, 60) to (X, A), i.e., 60 units to cell (X, A). The demand for the first column is satisfied. The allocation is shown in the following table.

Table (3.1)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20 60	22	17	4	~~120~~ 60
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	~~60~~	40	30	110	240

Now we move horizontally to the second column in the first row and allocate 40 units to cell (X, B). The demand for the second column is also satisfied.

Table (3.2)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20 60	22 40	17	4	~~120~~ ~~60~~ 40
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	~~60~~	~~40~~	30	110	240

Proceeding in this way, it was observed that $(X, C) = 20$, $(Y, C) = 10$, $(Y, D) = 60$, $(Z, D) = 50$. The resulting feasible solution has been shown in the following table.

Table (3.3)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20 60	22 40	17 20	4	~~120~~
Y	24	37	9 10	7 60	~~70~~
Z	32	37	20	15 50	~~50~~
Demand	~~60~~	~~40~~	~~30~~	~~110~~	240

Here, number of retail shops (n) = 4, and number of plants (m) = 3.

Number of basic variables = $m + n - 1 = 3 + 4 - 1 = 6$.

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

Initial basic feasible solution $= (20\times 60) + (22\times 40) + (17\times 20) + (9\times 10) + (7\times 60) + (15\times 50).$

$= \text{Ks}3680.$

4. Matrix Minimum Method

Matrix minimum (Least Cost) method is a method for computing a basic feasible solution of a transportation problem, where the basic variables are chosen according to the unit cost of transportation. This method is very useful because it reduces the computation and the time required to determine the optimal solution.

4.1 Example

Consider transformation Table (1.1).

Plant	Retail Shop				Supply
	A	B	C	D	
X	20	22	17	4	120
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	60	40	30	110	240

It was observed that $c_{14} = 4$, which is the minimum transportation cost. So $x_{14} = 110$. The demand for the fourth column is satisfied. The allocation is shown in the following table.

Table (4.1)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20	22	17	$\boxed{4}$ 110	~~120~~
Y	24	37	9	7	70
Z	32	37	20	15	50
Demand	60	40	30	~~110~~	240

Now it was observed that $c_{23} = 9$, which is the minimum transportation cost, so $x_{23} = 30$

Table (4.2)

Plant	Retail Shop				Supply
	A	B	C	D	
X	20	22	17	$\boxed{4}$ 110	~~120~~ 10
Y	24	37	$\boxed{9}$ 30	7	~~70~~ 40
Z	32	37	20	15	50
Demand	60	40	~~30~~	~~110~~	240

Proceeding in this way, it was founded that $x_{11} = 10$, $x_{21} = 40$, $x_{31} = 10$, $x_{32} = 40$. The resulting feasible solution has been shown in the following table.

Table (4.3)

Plant	Retail Shop				Supply
	A	B	C	D	
X	$\boxed{20}$ 10	22	17	$\boxed{4}$ 110	~~120~~
Y	$\boxed{24}$ 40	37	$\boxed{9}$ 30	7	~~70~~
Z	$\boxed{32}$ 10	$\boxed{37}$ 40	20	15	~~50~~
Demand	~~60~~	~~40~~	~~30~~	~~110~~	240

Number of basic variables = $m + n - 1$ = 3 + 4 – 1 = 6.

Initial basic feasible solution $= (4\times110)+(9\times30)+(20\times10)+(24\times40)+(32\times10)+(37\times40).$

$= \text{Ks}3670.$

Conclusion

To be able to use transportation modeling methods, managers need to know and understand complete transportation flow between their goods suppliers and buyers with current capacity and demands. The second principle of the transportation modeling is to use the right costs minimization approach to find a basic solution of a problem which will help to optimize the cost in the easiest and short–time way. The transportation modeling methods can be used in different sort of business areas with many suppliers, buyers and different quantities.

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